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ON THE PRACTICAL DETERMINATION OF HEIGHT FROM UPPER-AIR DATA

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Shaw, Keefer, Refsdal and others (**1**) have shown that geopotential height is represented on the tephigram by an area, and that an isentropic atmosphere XY (figure 1), equal in geopotential height to a given atmosphere AB, may be constructed by so placing the line XY on the tephigram that the area XAZ is equal to the area YBZ. The effect of moisture is normally negligible, but may be allowed for, if desired, by substituting virtual temperature for actual temperature in drawing the curve of state AB.

In an isentropic atmosphere the lapse rate is approximately constant and equal to 9.86° C. per kilometer (**2**) or 3° C. per 1,000 feet. The height of the atmosphere AB, in thousands of feet, is therefore equal to one-third of the temperature difference (in ° C.) between X and Y. This method of height determination, which gives values of a high degree of accuracy, is applicable to any energy diagram.

When the curve of state AB is irregular, the placing of the line XY by eye-estimation of the equality of the areas XAZ and YBZ may be a matter of considerable difficulty. A small error in the position of XY leads, however, to no appreciable error in the calculated height. If T_x and T_y are the temperatures at X and Y, respectively, θ the potential temperature (in degrees absolute) of the isentropic atmosphere XY, and p_0 and p the limiting pressures (fig. 1), then the height of XY in feet is

$$1{,}000\frac{T_x-T_y}{3},\quad \text{or}\ \frac{1{,}000}{3}\theta\left\{\left(\frac{p_0}{1{,}000}\right)^{0.288}-\left(\frac{p_1}{1{,}000}\right)^{0.288}\right\}.$$

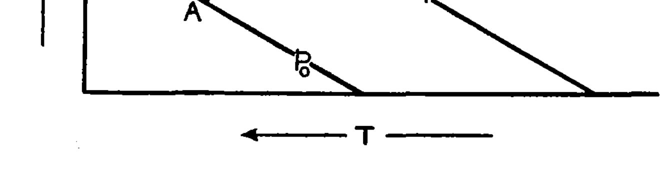

Fig. 1. Determination of Height on the Tephigram.

Hence the height of an isentropic atmosphere, between given pressure levels, is directly proportional to its potential temperature. An error of x° C. in the potential temperature of the equivalent isentropic atmosphere XY will therefore lead to an error of $\frac{x}{\theta}$ in the calculated height. For a mean position on the tephigram we may take $\theta=300°$ A; thus an error of 3° C. of potential temperature in the position of XY will cause an error of only 1 percent in the calculated height.

The use of a transparent scale with an engraved straight line facilitates the correct placing of XY; but the addition of a fixed scale of height reduces somewhat the accuracy of the method, owing to variation in the dimensions of the tephigram, particularly with humidity (**3**).

In practice, it is usually sufficient to estimate the position of the point Y. One-third of the difference (in °C.) between the potential temperature and the actual temperature at Y gives the height of B above the 1,000 mb. level in thousands of feet. A correction for the difference between the ground pressure and 1,000 mb. is then made by multiplying this difference by 30 and subtracting 10 percent.

A simple and speedy rule for the approximate determination of height has been formulated by E. Gold (**4**). Although originally intended for application to the tephigram, it can be used equally well in the absence of a diagram to determine the height of any point at which the potential temperature and actual temperature are known. Adapted for use with the centigrade scale, it reads: Take the difference between the potential temperature and actual temperature (in degrees centigrade) at the level of which the height is required; multiply by 2 and subtract 10 percent; again multiply by 2 and subtract 10 percent; this gives the value of the height above the 1,000 mb. level in hundreds of feet. The correction of the difference between the ground pressure and 1,000 mb. may be made as before.

It is clear that Gold's rule consists in multiplying $\theta_B - T_B$ by 324, where θ_B and T_B are the potential temperature and the actual temperature, respectively, at the point B whose height is required (fig. 1). A simple calculation shows that this process is equivalent to taking the height of the given point B as equal to that of an isentropic atmosphere whose potential temperature is $\frac{36}{37}\theta_B$ i. e. that of B reduced by $\frac{1}{37}\theta_B$, or 7–10° C. for the range of potential temperature provided on the diagram. The accuracy with which height is given by the rule depends on how closely this pseudo-equivalent isentropic atmosphere coincides with the true equivalent isentropic atmosphere XY, determined by the equal-area method, each 3°C. difference of potential temperature representing an error of 1 percent.

By considering different types of temperature distribution in the free air, it will be seen that:

(1) When applied to an isentropic atmosphere, Gold's rule leads to a figure which is 2.8 percent below the true height.

(2) The error is usually about 2 percent or less when the rule is applied to the lower levels of an average curve.

(3) The error is normally of the order of 1 percent at the higher levels (5,000–20,000 feet) of an average curve.

(4) When applied to the upper levels of a very stable curve (e. g. one featuring an extensive inversion), the rule leads to an overestimation of the height which may amount to 4 percent or more in an extreme case. The formula is least accurate when applied to the upper levels of such a curve.

From the fact mentioned above, that in an average situation the percentage error is greatest in the lowest levels, it follows that the absolute error is small at all heights in such a situation, and is usually of the order of 100–200 feet.

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AN EVALUATION OF THE BERGERON-FINDEISEN PRECIPITATION THEORY

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[Weather Bureau, Washington, May 1939]

The fundamental concept of the Bergeron-Findeisen precipitation theory was advanced by T. Bergeron (**1**) in 1935. As then formulated, it asserted that, disregarding some rather exceptional cases, the necessary condition for the formation of drops large enough to produce rain of any considerable intensity is that subfreezing temperatures exist in the cloud layer from which the rain descends. Findeisen (**2**) (**3**) has recently amplified this theory by introducing Wegener's postulate as to the existence of two kinds of nuclei—condensation nuclei and sublimination nuclei—on which the water vapor of the earth's atmosphere may respectively condense and sublime. The process thus amplified may be briefly described as follows:

Assuming that the dew-point of a mass of air is higher than the freezing point of water and that the mass of air contains both condensation nuclei (which are generally assumed to be ominipresent) and sublimation nuclei, let it be supposed that it is being cooled by any process or combination of processes. Under these conditions condensation will first take place on the condensation nuclei until the point is reached where the vapor pressure exerted by the sublimation nuclei is less than the vapor pressure exerted by the water droplets—this latter point, as will be shown later, seeming to be, in some cases at least, not far below the temperature of freezing. After this point is reached, any further cooling will cause the water vapor of the atmosphere to sublime on the sublimation nuclei and, at the same time, to be replenished by evaporation from the liquid drops. These latter processes will cause the resulting ice particles to become so large that they acquire a considerable rate of fall with respect to the water droplets, and, in their descent, they will continue to grow, not only by the evaporation-sublimation transfer of water from the surrounding water drops, but also by overtaking and coalescing with such drops as may happen to be in their path of fall. Since their size will not be limited by their rate of fall, these ice pellets can become quite large in the subfreezing layers of the cloud. When they encounter temperatures above the freezing point they will begin to melt and, if the resulting water drops are larger than the maximum raindrop size, they will break up into smaller drops—thus reaching the ground as rain.[2]

[2] If no sublimation nuclei had been present, under the circumstances assumed above, the continuance of the cooling would have resulted only in increasing the size of the cloud droplets—the cloud particles thus continuing to exist in the form of undercooled liquid drops. That this latter process cannot lead to the formation of precipitation was, however, shown by Bergeron by a series of simple calculations and considerations presented in his original paper (4).

Neither Bergeron nor Findeisen claim that the presence of subfreezing temperatures and sublimation nuclei is always necessary for the formation of precipitation. Findeison points out that if the humidities between the cloud layer and the ground are high enough, the cloud elements themselves may become sufficiently large to reach the ground as light rain or drizzle. Bergeron says that there are two other processes which may give rise to even heavy precipitation. The first process is instigated by what he calls the Reynolds effect in which those elements at the top of the cloud are cooled by radiation with a consequent reduction in the vapor pressure of the droplets so cooled and an increased condensation on them. These droplets thus acquire a size which is sufficient to cause them to fall through the lower part of the cloud and to thereby collide with the smaller and more slowly falling droplets, thus creating the observed rain. Bergeron points out, however, that in order to obtain heavy rain by this process, the cloud must have a great vertical thickness. Moreover, this process cannot set in unless some part of the cloud top is shielded from the sun's radiation.

The second explanation which Bergeron gives for the occurrence of heavy rain without subfreezing temperatures is that the electric field in the region may become so great that a coalescence of the cloud droplets is brought about by the induction of electrical charges within the droplets. In discussing the potentialities of this effect, he simultaneously considers the possibilities of the coalescence of droplets of equal size due to hydrodynamical attraction. He apparently discards hydrodynamical attraction in favor of that due to electrostatic induction on the basis of a set of computations made in "Physikalische Hydrodynamik" by V. Bjerknes, J. Bjerknes, H. Solberg, and T. Bergeron (**6**). Köhler, however, has pointed out (**7**) that the results of Bjerknes' electrostatic induction computations are too large by a factor of 10^4. It also appears that the results of his hydrodynamical computations are too small by a factor of 10^2. When these two errors are considered along with the fact that the electric field of the earth's atmosphere has been found to decrease rapidly with height above an altitude of four or five kilometers (8), it would seem that, assuming the remainder of the calculations to be correct, the effects of any electrostatic induction attractions which may be present must be subordinated to the hydrodynamical attraction effects in attempting to account for the formation of precipitation.